ENERGY APPROACH TO THE DESCRIPTION OF FATIGUE CRACK GROWTH IN A NON-UNIAXIAL STRESS STATE

V. V. Bolotin

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An energy approach for describing fatigue crack growth is developed on the basis of the following postulate [1]: A growing fatigue crack becomes nonequilibrium in the Griffith sense at least once in a loading cycle if resistance to crack growth is calculated with allowance for the damages accumulated during previous loading. This approach is extended to the case of a non-uniaxial stress state with the inclusion of the phenomena of arrest, irregular growth, rotation, and branching of cracks. The general structure of the equations for a certain crack growth rate in a non-uniaxial stress state is discussed.

1. Fatigue crack growth occurs under conditions of interaction of the crack with the process of accumulation of scattered damages in the material. In fact, there are grounds for suggesting that damage accumulation in front of cracks is the main mechanism governing the fatigue crack growth rate. This notion has been expressed repeatedly in more or less explicit fashion (see, e.g., the survey articles in [2]). However, specific mathematical models based on this idea have been developed only for low-cycle fatigue — a phenomenon into the description of which the additional scale of length (size of the plastic zone) enters naturally. A survey and a critical discussion of these models can be found in [3]. Models of fatigue-crack growth which take into account the accumulation of dislocations at the front of cracks have also been proposed [4]. Usually, semi-empirical equations are used to describe fatigue-crack growth [5]. The present study elaborates on the model of high-cycle fatigue proposed in [1] and based on concepts of continuum mechanics. The model employs a generalization of the energy approach to fracture mechanics.

We will refer to the state of a body with cracks as subequilibrium and as nonequilibrium after Griffith if the conditions $\delta I = 0$, $\delta I < 0$ and $\delta I > 0$ are satisfied, respectively. Here, δI is the Griffith variation of the total energy of the body—load system [1] taken with the opposite sign. If the aggregate of cracks in the body is specified to within m generalized coordinates $l_1, \ldots, l_m$ (for example, characteristic dimensions of the cracks), then we can take the following for the expression of δI

$$\delta I = \sum_{j=1}^{m} (G_j - \Gamma_j)\, \delta l_j. \tag{1.1}$$

Here, G_j are the generalized forces advancing the cracks (analogs of liberated Irwin energy; Γ_j are generalized resistance forces (analogs of critical values of liberated energy).

We will examine the process of cyclic loading of a body specified by a vector $\mathbf{s}(t)$. We will designate the vector of the generalized coordinates $l_1, \ldots, l_m$ as $\mathbf{l}(t)$. We will use $\psi(t)$ to represent the vector of the damages accumulated at the fronts of the cracks. For an undamaged material, $\psi = 0$. We will formulate the postulate on fatigue crack growth as follows: A crack obtains an increment with respect to one of the generalized coordinates during a cycle if it becomes nonequilibrium with respect to this coordinate at least once during the cycle with the condition that the corresponding generalized resistance force is calculated with allowance for the damages accumulated during previous loading.

For analytical formulation of the postulate, let us examine a function of the values of the processes $\mathbf{l}(t)$, $\mathbf{s}(t)$, and $\psi(t)$ on the segment $(t_{n-1}, t_n]$ corresponding to the n-th cycle:

$$H_j(n) = \sup_{t_{n-1} < t \leqslant t_n} \{G_j[\mathbf{l}(t), \mathbf{s}(t), \psi(t)] - \Gamma_j[\mathbf{l}(t), \mathbf{s}(t), \psi(t)]\} \quad (j = 1, \ldots, n). \tag{1.2}$$

When $H_j(n) < 0$, generalized coordinate l_j does not increase, and damage accumulation proceeds at the front of the stopped crack. When $H_j(n) > 0$, this means that the crack becomes nonequi-

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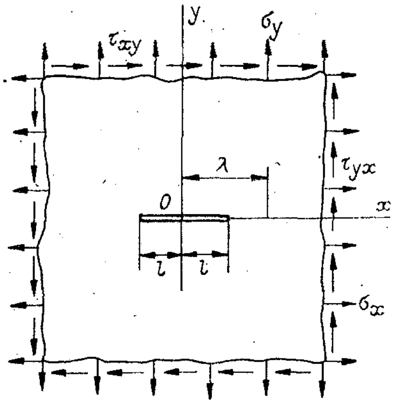

Fig. 1

librium at least once during the n-th cycle and, thus, unstable as well. The new dimension $l_j(t_n)$ is found from the energy balance condition

$$\int_{l_j(t_{n-1})}^{l_j(t_n)} G_j\left[\lambda, \mathbf{s}(t_j^*), \boldsymbol{\varphi}(\lambda, t_j^*)\right] d\lambda = \int_{l_j(t_{n-1})}^{l_j(t_n)} \Gamma_j\left[\lambda, \mathbf{s}(t_j^*), \boldsymbol{\varphi}(\lambda, t_j^*)\right] d\lambda, \tag{1.3}$$

where $\boldsymbol{\varphi}(\lambda, t)$ is the vector-function of the damages on the continuation of the front of the growing crack; t_j^* is the moment of time corresponding to attainment of the supremum in the right side of Eq. (1.2). On the crack fronts $\lambda = \mathbf{l}(t)$.

Equation (1.3) is applicable if the crack grows with respect to one of the generalized coordinates during a cycle. When growth occurs over several coordinates, it is sufficient to require that the dynamic processes decay during the intervals between adjacent moments of time t_j^* and t_k^*. If the newly found values correspond to a stable system of cracks lying within the body, then further growth is possible. For subsequent study, we will need to examine the signs of the function $H_j(n + 1)$, construct equations of type (1.3), etc.

A typical pattern of fatigue crack growth consists of the following. After a sudden increase in crack size to the dimension $l_j(t_n)$, it becomes subequilibrium due to the attainment by its front of a section with relatively little damage. Thus $H_j(n + 1) < 0$. It is necessary to determine the number of cycles before which the subequilibrium condition will be violated. Then a sudden crack growth will occur again, etc. Thus, we arrive at a natural explanation for the stria on the surface of a fatigue fracture.

To obtain a closed system of equations, it is necessary to take equations describing damage accumulation on the continuation of cracks. We will construct a finite-difference functional equation in the vector-function $\boldsymbol{\varphi}(\lambda, t)$:

$$\boldsymbol{\varphi}(\lambda, t_n) - \boldsymbol{\varphi}(\lambda, t_{n-1}) = \underset{t=t_{n-1}}{\overset{t=t_n}{\Phi}} \{\lambda, \mathbf{l}(t), \mathbf{s}(t), \boldsymbol{\varphi}(\lambda, t)\}. \tag{1.4}$$

Here, $\Phi\{\cdot\}$ is some specific functional of the loading history and damage accumulation on the segment of the n-th cycle. In describing high-cycle fatigue, the number of cycles n is usually treated as a continuous argument. If the load parameters change slowly in the transition from one cycle to another, then the vector-function $\mathbf{s}(t)$ can be replaced by the continuous vector-function $\mathbf{s}(n)$, the components of which include extreme values of the loading parameters and, if necessary, the frequency of the cycles. In this case, $\mathbf{l}(n)$ and $\boldsymbol{\psi}(n)$ are also slowly changing functions of n. Applying the mean-value theorem to (1.3), we arrive at the system of equations

$$G_j[\mathbf{l}(n), \mathbf{s}(n), \boldsymbol{\psi}(n)] = \Gamma_j[\mathbf{l}(n), \mathbf{s}(n), \boldsymbol{\psi}(n)] \quad (j = 1, \ldots, m). \tag{1.5}$$

This means that slowly growing fatigue cracks are almost no different from Griffith equilibrium cracks with allowance for the damages accumulated at their fronts [1]. In (1.5)

$$\boldsymbol{\psi}(n) \equiv \boldsymbol{\varphi}[\mathbf{l}(n), n], \tag{1.6}$$

while for the vector-function $\boldsymbol{\varphi}(\lambda, n)$ instead of (1.4) we have the equation

$$\partial\boldsymbol{\varphi}(\lambda, n)/\partial n = \Phi[\lambda, \mathbf{l}(n), \mathbf{s}(n), \boldsymbol{\varphi}(\lambda, n)]. \tag{1.7}$$

When there are important changes in the loading process, such as with a sudden increase in the load of a change in the type of nominal stress state, it is necessary to revert to the more general equations (1.3-1.4).

2. Let us examine a fatigue crack in an infinite body in a plane stress state with nominal stresses σ_x, σ_y, and τ_{xy} (Fig. 1). The crack length is $2l$. We assume that the crack propagates without branching and rotations, i.e., its continuation (extension) lies on the Ox axis. Due to the symmetry of the problem, we will examine the crack as a single-parameter crack specified to within l. Here, we are considering that except for the regions in which scattered damages have accumulated, the material has properties corresponding to linear fraction mechanics. We find the generalized force from the Irwin formula [2]

$$G = (1-\nu^2)\left(K_1^2 + K_2^2\right)/E, \quad K_1 = \sigma_y (\pi l)^{1/2}, \quad K_2 = \tau_{xy} (\pi l)^{1/2}, \tag{2.1}$$

ignoring the contribution of the regular components of the stress field to the liberated energy. We will also ignore the effect of the cumulative damages, so that $G = G(l, \sigma_x, \tau_{xy})$.

We will distinguish two damage measures, constituting the vector-function $\varphi(\lambda, t)$. The measure φ_1 accounts for normal-rupture microcracks oriented along the Ox axis. These cracks are formed mainly as a result of the cyclic change in stresses σ_y. The measure φ_2 accounts for shear microcracks oriented in the same direction. The accumulation of these cracks is determined mainly by the magnitude of the shear stresses τ_{xy}. We will write the damage accumulation equations (1.4) in a form consistent with the semiempirical equations used to describe fatigue damages [6]:

$$\begin{aligned} \varphi_1(\lambda, t_n) - \varphi_1(\lambda, t_{n-1}) &= (\Delta\sigma - \Delta\sigma_{th})^{m_1}/\sigma_f^{m_1}, \\ \varphi_2(\lambda, t_n) - \varphi_2(\lambda, t_{n-1}) &= (\Delta\tau - \Delta\tau_{th})^{m_2}/\tau_f^{m_2}. \end{aligned} \tag{2.2}$$

Here $\Delta\sigma(\lambda, n)$ and $\Delta\tau(\lambda, n)$ are the amplitudes of the rupture (tensile) and shear stresses, respectively, during the n-th cycle at a point lying on the crack extension; σ_f and τ_f are characteristics of the material describing its resistance to damage accumulation; $\Delta\sigma_{th} \geqslant 0$, $\Delta\tau_{th} \geqslant 0$ are threshold values at which damages begin to accumulate; m_1 and m_2 are constant exponents. Equations (2.2) are satisfied when $\Delta\sigma > \Delta\sigma_{th}$, $\Delta\tau > \Delta\tau_{th}$. If one of these inequalities is not satisfied, then the right side in the corresponding equation (2.2) must be set equal to zero.

The dependence of the stresses σ and τ on the distance $\lambda - l$ from the crack front should give finite values of the stress concentration factors on the front and describe an asymptotic approximation to the nominal stresses when $\lambda \gg l$. Here, naturally, there should appear a certain new length scale $\rho \ll l$, which is an important element of the theory being developed. The parameter ρ can take values from a wide range, beginning from the characteristic grain-boundary thickness to the characteristic grain size. We choose it so that the typical stress concentration factors on the boundaries have the order $(l/\rho)^{1/2}$. This condition is satisfied by the following expressions

$$\sigma = K_1[\pi h_1(\lambda)]^{-1/2}, \quad \tau = K_2[\pi h_2(\lambda)]^{-1/2}, \tag{2.3}$$

where the functions $h_{1,2}(\lambda)$ have the form

$$h_{1,2}(\lambda) = \begin{cases} \chi_{1,2}\rho, & l \leqslant \lambda \leqslant l + \rho, \\ \chi_{1,2}\rho + (\lambda - l - \rho)(l/\lambda), & \lambda > l + \rho. \end{cases} \tag{2.4}$$

The form factors $\chi_{1,2}$ depend on the configuration of the crack front.

In the case of high-cycle fatigue, the crack moves very slowly. This makes it possible to change over from Eqs. (2.2) to the continuous approximation. Here, we obtain the following equation for the damage in the structural element (Fig. 2) closest to the front

$$d\psi_1/dn = (\Delta K_1 - \Delta K_{th,1})^{m_1}/K_{f1}^{m_1}, \quad d\psi_2/dn = (\Delta K_2 - \Delta K_{th,2})^{m_2}/K_{f2}^{m_2}, \tag{2.5}$$

where we have used the notation for the amplitudes of the stress intensity factors $\Delta K_1 = \Delta\sigma_y (\pi l)^{1/2}$ and $\Delta K_2 = \Delta\tau_{xy}(\pi l)^{1/2}$. We also expressed characteristics of the resistance of the material to damage accumulation in terms of the stress intensity factors

$$\begin{aligned} K_{f1} &= \sigma_f(\pi\chi_1\rho)^{1/2}, \quad K_{f2} = \tau_f(\pi\chi_2\rho)^{1/2}, \\ \Delta K_{th,1} &= \Delta\sigma_{th}(\pi\chi_1\rho)^{1/2}, \quad \Delta K_{th,2} = \Delta\tau_{th}(\pi\chi_2\rho)^{1/2}. \end{aligned} \tag{2.6}$$

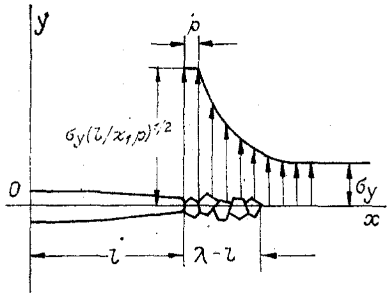

Fig. 2

For the generalized force Γ we have the relation

$$\Gamma = \Gamma_0 g(\psi_1, \psi_2), \tag{2.7}$$

where Γ_0 is the force associated with resistance to advance of the crack into the undamaged material; $\Gamma_0 = 2\gamma_0$ (γ_0 is the unit fracture work). The function $g(\psi_1, \psi_2)$ should satisfy the conditions $g(0, 0) = 1$, $g(1, 0) = g(0, 1) = 0$. This means that the resistance to crack advance vanishes even if only one of the damage measures reaches the limiting value of unity. Henceforth, we will examine the function

$$g(\psi_1, \psi_2) = [1 - (\psi_1 + \psi_2)^\alpha]^\beta, \tag{2.8}$$

where $\alpha > 0$, $\beta > 0$.

A qualitative picture of crack growth in shown in Fig. 3a, where the relation $G(\lambda)$ is taken in accordance with Eq. (2.1) with maximum values of the nominal stresses independent of n. The critical (after Griffith) crack size l_* satisfies the condition $G(\lambda) = \Gamma_0$. Let $G < \Gamma$ at the beginning of the n-th cycle. Damage accumulation on a segment of length $l(t_{n-1}) \leqslant \lambda \leqslant l(t_{n-1}) + \rho$ occurs until the moment of time t_n, when the equality $G = \Gamma$ is reached on the front. We find the new dimension $l(t_n)$ from condition (1.3). It corresponds to equality of the areas of the hatched triangles in Fig. 3. The increment in crack length $\Delta l = l(t_n) - l(t_{n-1}) > \rho$. Then the process is repeated. The fatigue crack as a whole becomes unstable when the size $l_{f*} < l_*$ is reached, since for all $\lambda > l_{f*}$ we have the inequality $G > \Gamma$.

The supposition that the stresses and damages within an element of the size ρ must be the same is not obligatory. This is illistrated by Fig. 3b. Crack advance occurs at the moment the equality $G(\lambda + \rho) = \Gamma$ is satisfied.

To analytically describe this variant, it is sufficient to replace $l(t_{n-1})$ by $l(t_{n-1}) + \rho$ in (1.2). Obviously, Eqs. (1.5) for quasiequilibrium fatigue crack growth remains the same.

3. We will derive approximate differential equations describing slow crack growth. We will evaluate the duration of the intersection of the crack front by a segment of length ρ from the formula $\Delta n \approx \rho (dl/dn)^{-1}$. Ignoring the damages accumulated until the front reaches the structural element, we obtain

$$\psi_{1,2}(n) = \rho (dl/dn)^{-1} \Phi_{1,2}(n). \tag{3.1}$$

Here, the functions $\Phi_{1,2}(n)$ coincide with the right sides of Eqs. (2.5). The quasiequilibrium condition, with allowance for (2.4), (2.7), (2.8), and (3.1), leads to the equation

$$\frac{dl}{dn} = \rho \frac{(\Delta K_1 - \Delta K_{\text{th},1})^{m_1}/K_{f1}^{m_1} + (\Delta K_2 - \Delta K_{\text{th},2})^{m_2}/K_{f2}^{m_2}}{\left[1 - (K_1^2 + K_2^2)_{\max}^{1/\beta}/K_{1c}^{2/\beta}\right]^{1/\alpha}}. \tag{3.2}$$

Here, $K_{1c}^2 = E\Gamma_0(1 - \nu^2)^{-1}$, i.e., K_{1c} is the critical value of the stress intensity factor for the undamaged material. In regard to a classical Griffith crack, an equation of type (3.2) is obtained on the basis of the energy approach taken in [1].

The form of the right side of (3.2) depends considerably on the particular assumptions made with respect to the structure of the right sides of Eqs. (2.2) and (2.5), as well as on the form of the functions $g(\psi_1, \psi_2)$ in Eq. (2.7). However, the structure of Eq. (3.2) remains general, with a very broad range of suppositions. The numerator in the right side is determined by the rate of accumulation of fatigue microdamages at the crack front, while the denominator contains the ratio of the maximum amount of released energy to its critical value for

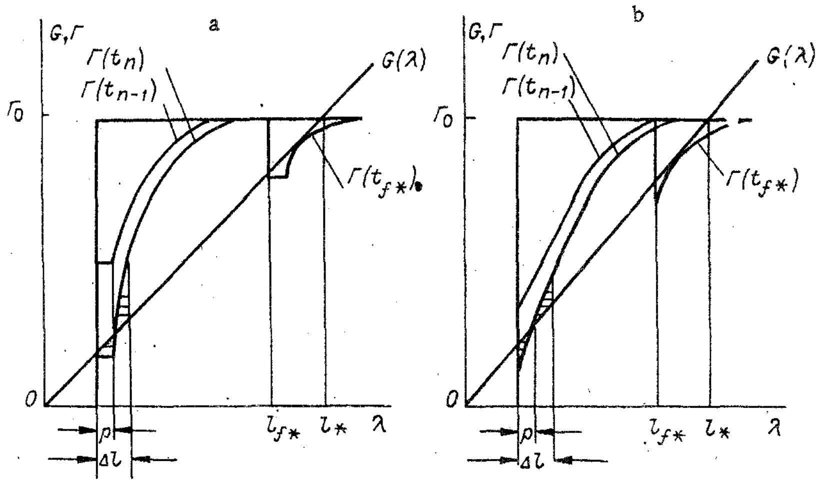

Fig. 3

the undamaged material. The maximum of liberated energy corresponds to the condition of the supremum in (1.2).

4. Rotations and branchings of fatigue cracks are among the most difficult and least studied aspects of fracture mechanics. The empirical data on the direction of fatigue crack growth in a complex stress state is not completely clear and is often contradictory [7]. Fatigue cracks exhibit a tendency to branch and grow in a zig-zag manner even in uniaxial tension. A discussion of the problem can be found in [8-11]. The application of analytical methods to the problem of crack propagation is made difficult by the shortage of information on the stress distribution in the vicinity of cracks of complex configuration. One of the few exceptions here is the zig-zag crack. Stress intensity factors near small branches, i.e., when $\lambda - l \ll l$, were calculated in [8, 9]. The most reliable data was obtained in [10] using a fairly cumbersome analytical and computational approach. We will make use of this data to analyze the problem of rotations and branching of an initially planar crack (Fig. 4).

For a crack the front of which can rotate through an angle θ, where $-\pi \leqslant \theta \leqslant \pi$, Eq. (1.5) takes the form

$$G[l(n), s(n), \theta] = \Gamma[l(n), \psi(n, \theta)]. \qquad (4.1)$$

Here, in regard to the above-examined model, the left side is taken to be independent of $\psi(n)$, while the right side is independent of $s(n)$. The generalized force G depends explicitly on θ, while the generalized force Γ depends on θ through the damage measure $\psi(n, \theta)$. We will calculate the left side of (4.1) from the Irwin formula (2.1) with allowance for the dependence of the stress intensity factor on the angle θ:

$$G = [(1-\nu^2)/E]\{[K_1 f_{11}(\theta) + K_2 f_{12}(\theta)]^2 + [K_1 f_{21}(\theta) + K_2 f_{22}(\theta)]^2\}. \qquad (4.2)$$

Here, K_1 and K_2 are the stress intensity factors with $\theta = 0$, i.e., determined in accordance with (2.1). Figure 5 shows graphs for the functions $f_{jk}(\theta)$ constructed using the data from [10]. The nominal stresses σ_x and the regular components of the stress field do not enter into (4.2), which is equivalent to some additional assumption corresponding to concepts in linear fracture mechanics. If the quantity $K_1 f_{11}(\theta) + K_2 f_{12}(\theta)$ turns out to be negative, then it should be set equal to zero.

To describe damages at the crack front, we generalize Eqs. (2.2). We will consider the dependence of damage measures φ_1 and φ_2 on the angle θ. Here, we assume that an increment in the measure φ_1 depends on the amplitude $\Delta\sigma_\theta$ of the rupture stress on an area inclined at the angle θ. For the increment of the measure φ_2, we postulate similar dependences on the amplituce $\Delta\tau_{r\theta}$ of the shear stress on this area (see Fig. 4). From this

$$\begin{aligned} \varphi_1(\lambda, \theta, t_n) - \varphi_1(\lambda, \theta, t_{n-1}) &= (\Delta\sigma_\theta - \Delta\sigma_{th})^{m_1}/\sigma_f^{m_1}, \\ \varphi_2(\lambda, \theta, t_n) - \varphi_2(\lambda, \theta, t_{n-1}) &= (\Delta\tau_{r\theta} - \Delta\tau_{th})^{m_2}/\tau_f^{m_2}. \end{aligned} \qquad (4.3)$$

The characteristics of the material retain their former significance. For the stresses in the vicinity of the crack front, we take expressions consistent with Eqs. (2.3) and (2.4):

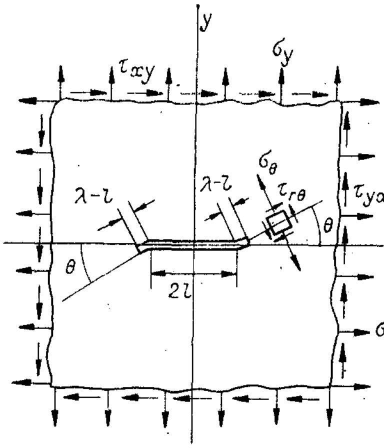

Fig. 4

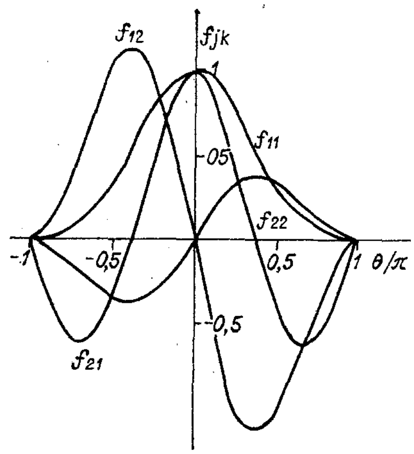

Fig. 5

$$\sigma_\theta = [K_1 g_{11}(\theta) + K_2 g_{12}(\theta)][\pi h_1(\lambda)]^{-1/2},$$
$$\tau_{r\theta} = [K_1 g_{21}(\theta) + K_2 g_{22}(\theta)][\pi h_2(\lambda)]^{-1/2}. \qquad (4.4)$$

We take the angle function $g_{jk}(\theta)$ in the form

$$g_{11} = \cos^3(\theta/2),\ g_{12} = (3/2)\cos(\theta/2)\sin\theta,$$
$$g_{21} = (1/2)\cos(\theta/2)\sin\theta,\ g_{22} = (1/2)(3\cos\theta - 1)\cos(\theta/2). \qquad (4.5)$$

The graphs of functions (4.5) are similar to the graphs of the functions $f_{jk}(\theta)$ shown in Fig. 5. Moreover, with a relatively high degree of accuracy (the greatest error on the segment $-\pi/2 \leqslant \theta \leqslant \pi/2$ is 10%), we can put $f_{jk}(\theta) \approx g_{jk}(\theta)$. Angle functions (4.5) were taken from the Williams formulas for the stress distribution near a two-dimensional mathematical slit [2, 5]. The stresses (4.4) coindice at $l \leqslant \lambda \leqslant l + \rho$ with the stresses calculated from the Williams formula if in the latter we take the polar radius $r = \rho/2$ (Fig. 6). Thus, the representation regarding the finiteness of the stresses at the front of a growing crack are consistent with the usual representations of linear fracture mechanics.

For slowly growing cracks, with allowance for Eqs. (4.4) and the notation (2.6), Eqs. (4.3) take the form

$$\partial\psi_1/\partial n = [\Delta K_1 g_{11}(\theta) + \Delta K_2 g_{12}(\theta) - \Delta K_{th,1}]^{m_1}/K_{f_1}^{m_1},$$
$$\partial\psi_2/\partial n = [|\Delta K_1 g_{21}(\theta) + \Delta K_2 g_{22}(\theta)| - \Delta K_{th,2}]^{m_2}/K_{f2}^{m_2}. \qquad (4.6)$$

The general representation (2.7) for the generalized resistance force Γ and its special case (2.8) remain unchanged, while the dependence on θ enters implicitly through the solution of Eqs. (4.6).

5. Let us discuss certain qualitative conclusions from the proposed theory. First we will examine a normal-rupture crack (Fig. 7). Let the material be such that only a normal-rupture microcrack facilitates the growth of a fatigue crack. Then in (4.6) we need to set $K_{f2} \to \infty$, and in place of (2.8) we take $\Gamma = \Gamma_0(1 - \psi_1^\alpha)^\beta$. Slow quasiequilibrium growth of such a crack is schematized in Fig. 7a. Curve 1 corresponds to the function G(θ, n), calculated from Eq. (4.2). Curve 1' corresponds to the function Γ(θ, n). The growing crack differs little from a Griffith equilibrium crack. As it grows, the angular dependence of the generalized force G(θ, n) is described by curves 2 and 3, while that of the generalized force Γ(θ, n) is described by curves 2' and 3'. The direction of crack growth remains the same, i.e., it corresponds to the angle $\theta_1 = 0$.

The picture changes significantly if fatigue crack growth occurs as a result of the accumulation of shear microcracks (Fig. 7b). For the generalized force G(θ, n), as before we have curve 1. We have curve 1' for the generalized forces Γ(θ, n). The Griffith equilibrium condition is reached when $\theta = \pm\theta_1$. This means that the crack has a tendency to branch at these angles and to grow in a zig-zag manner at these angles. Incidentally, the proposed theory does not take into account the change in the stress and damage fields due to secondary rotation or branching of the crack. In the general case, the damages at the crack front are comprised of normal-rupture and shear microcracks, while the angular dependence of the function Γ(θ, n) has the form 2' in Fig. 7b. The conditions are close to equilibrium on the seg-

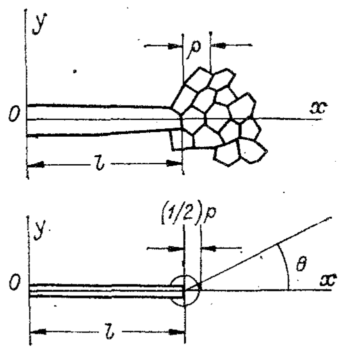

Fig. 6

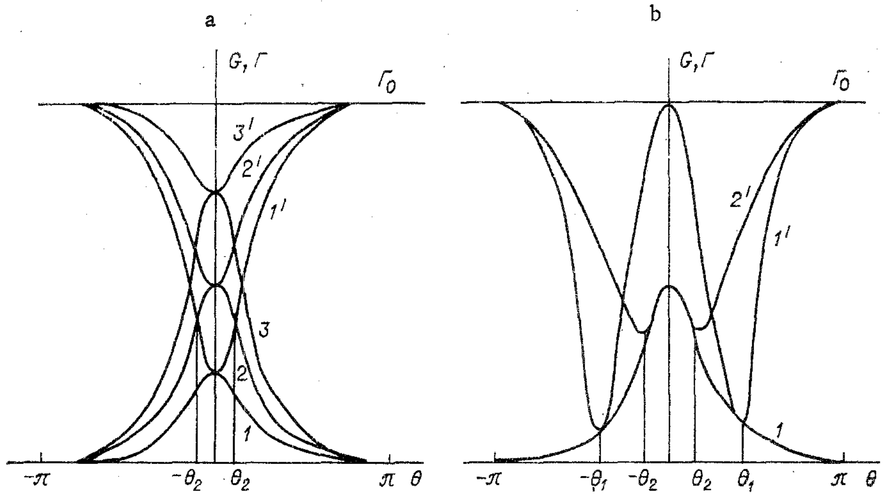

Fig. 7

ment $[-\theta_2, \theta_2]$. This means that all of the directions from this segment are of roughly equal probability. Thus, the element of unpredictability in crack branching is explained to a considerable extent by the interaction of damage accumulation mechanisms. In this situation, the mathematical expectation of the angle of propagation $\langle\theta\rangle = 0$.

The case of a complex stress state is illustrated in Fig. 8. Let the crack initially grow as a normal-rupture crack. This corresponds to curves 1 and 1'. Beginning with a certain cycle, the character of the stress state changes: For example, cyclic tension is replaced by cyclic shear. The angular dependence of the generalized force $G(\theta, n)$ is shown by curve 2 in Fig. 8a. Let the new loading regime be such that at first the crack remains subequilibrium. This continues until additional damages are accumulated at the front so that the angle function $\Gamma(\theta, n)$ takes the form shown by curve 2'. The crack begins to grow at the angle θ_2 (in this example, $\tau_{xy} < 0$).

Figure 8b shows a situation in which the crack becomes a nonequilibrium crack after the change in stress state, i.e., there appears a segment (θ'_2, θ''_2) on which $H(\theta, n) = G(\theta, n) - \Gamma(\theta, n) > 0$. Here, the crack grows in an abrupt manner, and it may even branch suddenly. The corresponding angles are distributed randomly on the segment (θ'_2, θ''_2). The main quantity which characterizes the probable angle distribution is the difference on the generalized forces $H(\theta, n)$. It is natural to assume that the probability density $f(\theta)$ has the form $f(\theta) = \text{const}\{1 - \exp[-\beta H(\theta)]\}$, where β is some positive constant. The most probable direction is that in which the function $H(\theta, n)$ reaches a maximum. We obtain similar results in the case when the stress level changes suddenly without a change in the stress state. For example, if the nominal stresses increase in normal rupture so that curve 1 of the generalized force $G(\theta, n)$ in Fig. 7a is replaced by curve 2, the angles of branching of the crack will be distributed of the segment $(-\theta_2, \theta_2)$, for which $H(\theta, n) > 0$. The most probable direction of crack propagation corresponds to the angle $\langle\theta\rangle = 0$.

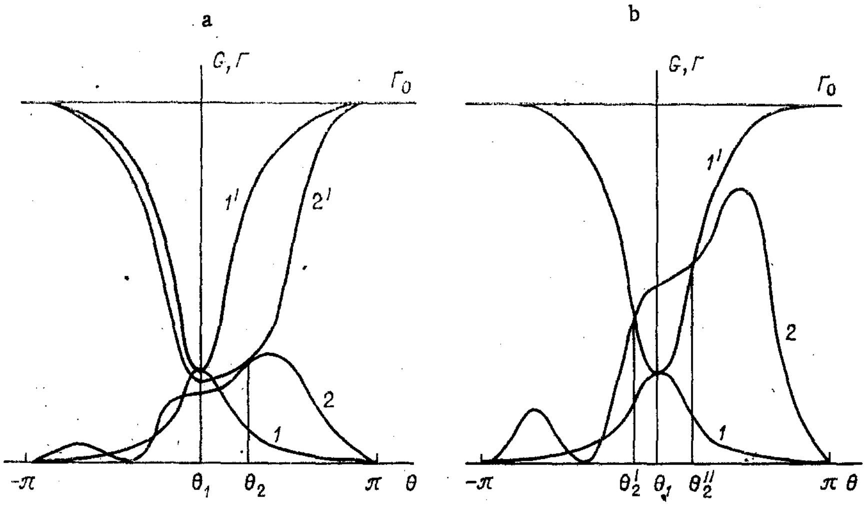

Fig. 8

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